

Newtonian Fractional-Dimension Gravity and Dark Matter

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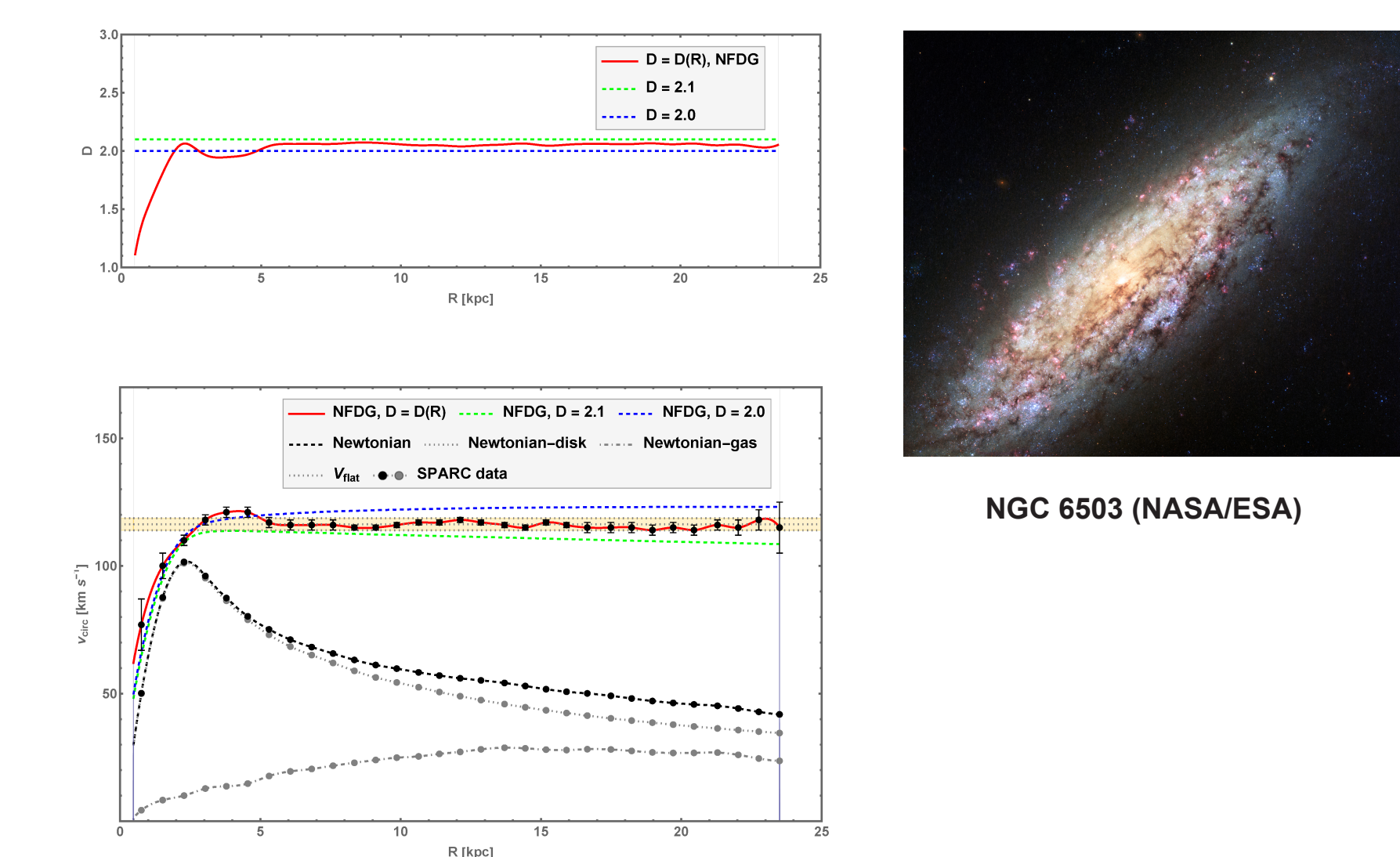


Introduction

Newtonian Fractional-Dimension Gravity (NFDG) is an alternative model of gravity, which was introduced in 2020 with the first three papers on the subject. The goal of NFDG is to model galaxy rotation curves without using the controversial Dark Matter (DM). This is done by assuming that galactic structures behave as fractal media, with an effective dimension which can be lower than the standard value $D = 3$, including also possible fractional, i.e., non-integer dimensions. This model is broadly based on the methods of fractional calculus and is connected with Modified Newtonian Dynamics (MOND). A fourth paper (2021) introduced a relativistic extension: Relativistic Fractional-Dimension Gravity (RFDG), which might be useful for astrophysical or cosmological applications. A fifth paper (2022) analyzed additional galaxies in relation with the External Field Effect (EFE). In a sixth paper (2023), NFDG was applied to some notable cases of galaxies with little or no dark matter, and to the case of the Bullet Cluster merger.

NGC 6503

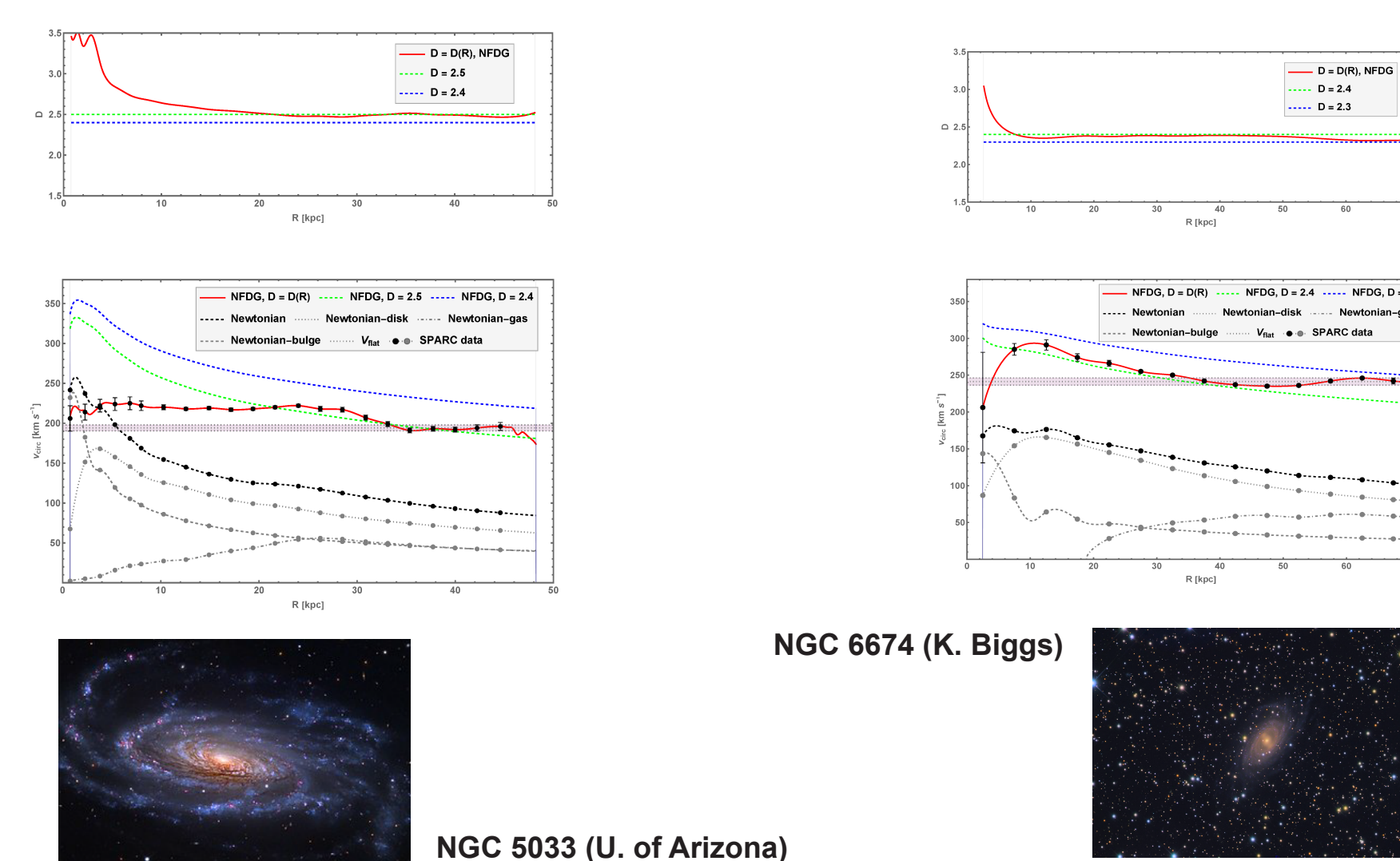
NFDG analysis of this disk-dominated galaxy: top panel shows the variable dimension $D(R)$ (red-solid curve), computed with NFDG models. Bottom panel: NFDG galaxy rotation curves compared with SPARC data. If this galaxy behaves as a fractal structure with dimension $D(R)$ following the red-solid curve in the top panel, then the resulting NFDG circular velocity curve in the bottom panel (red-solid) will perfectly fit the experimental data, without any need of dark matter.



Additional NFDG results

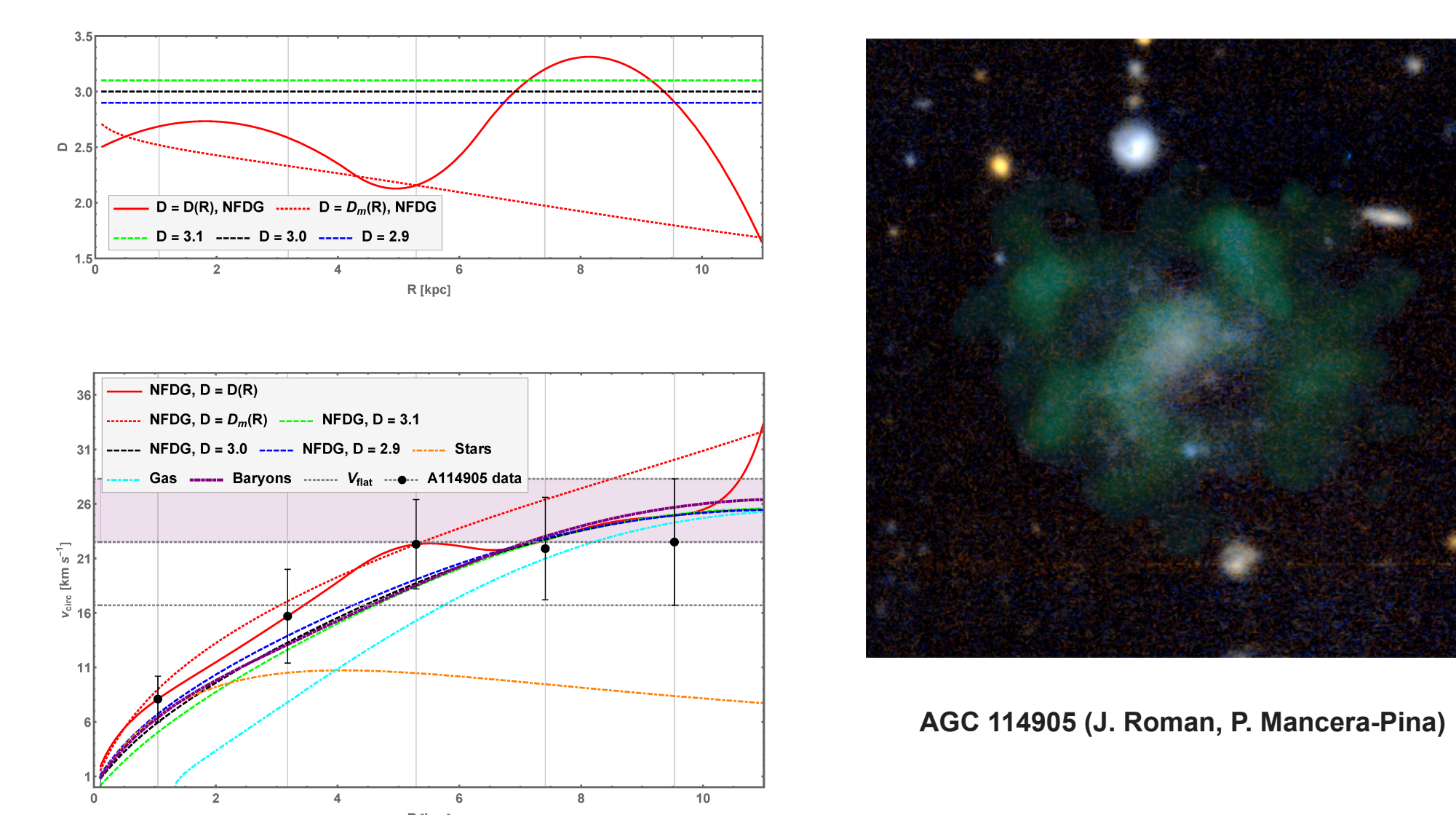
NGC 5033, NGC 6674, NGC 5055, and NGC 1090

Four more galaxies were studied with NFDG methods in a recent paper (2022), in relation to the so-called External Field Effect (EFE). While the NFDG theory does not imply the EFE, these four galaxies were modeled like the previous three galaxies and perfect fits to the galactic rotation curves were produced, as can be seen below.



Galaxies without DM

NFDG was also applied to some notable cases of galaxies with little or no dark matter (2023). In the case of the ultra-diffuse galaxy AGC 114905, NFDG methods effectively reproduced the observed rotation curve using a variable fractional dimension $D(R)$, as was done for previous galaxies (red-solid curves below). We obtain a variable dimension in the range $D = 2.2 - 3.2$, but our fixed $D = 3$ curve can also fit all experimental data, within error bars.



A brief review of NFDG

NFDG is based on a heuristic extension of Gauss's law for gravitation to a lower-dimensional space-time $D+1$, where $D \leq 3$ can become a non-integer space dimension. For a point-like mass m placed at the origin, the NFDG gravitational potential is:

$$\Phi_{NFDG}(r) = -\frac{2\pi^{1-D/2}\Gamma(D/2)Gm}{(D-2)l_0r^{D-2}}; D \neq 2$$

$$\Phi_{NFDG}(r) = \frac{2Gm}{l_0}\ln r; D = 2,$$

where G is the gravitational constant, r is a rescaled, dimensionless coordinate, and l_0 is an appropriate scale length. This potential is then generalized to model the spherical bulge and the cylindrical stellar disk and gas mass distributions of a galaxy. The space dimension $D = D(R)$ can be a function of the radial coordinate R in the galaxy disk plane, but we neglect its space derivatives in the computation of the NFDG gravitational field:

$$\mathbf{g}_{NFDG}(R) = -\frac{1}{l_0} \frac{d\Phi_{NFDG}(R)}{dR} \hat{\mathbf{R}}.$$

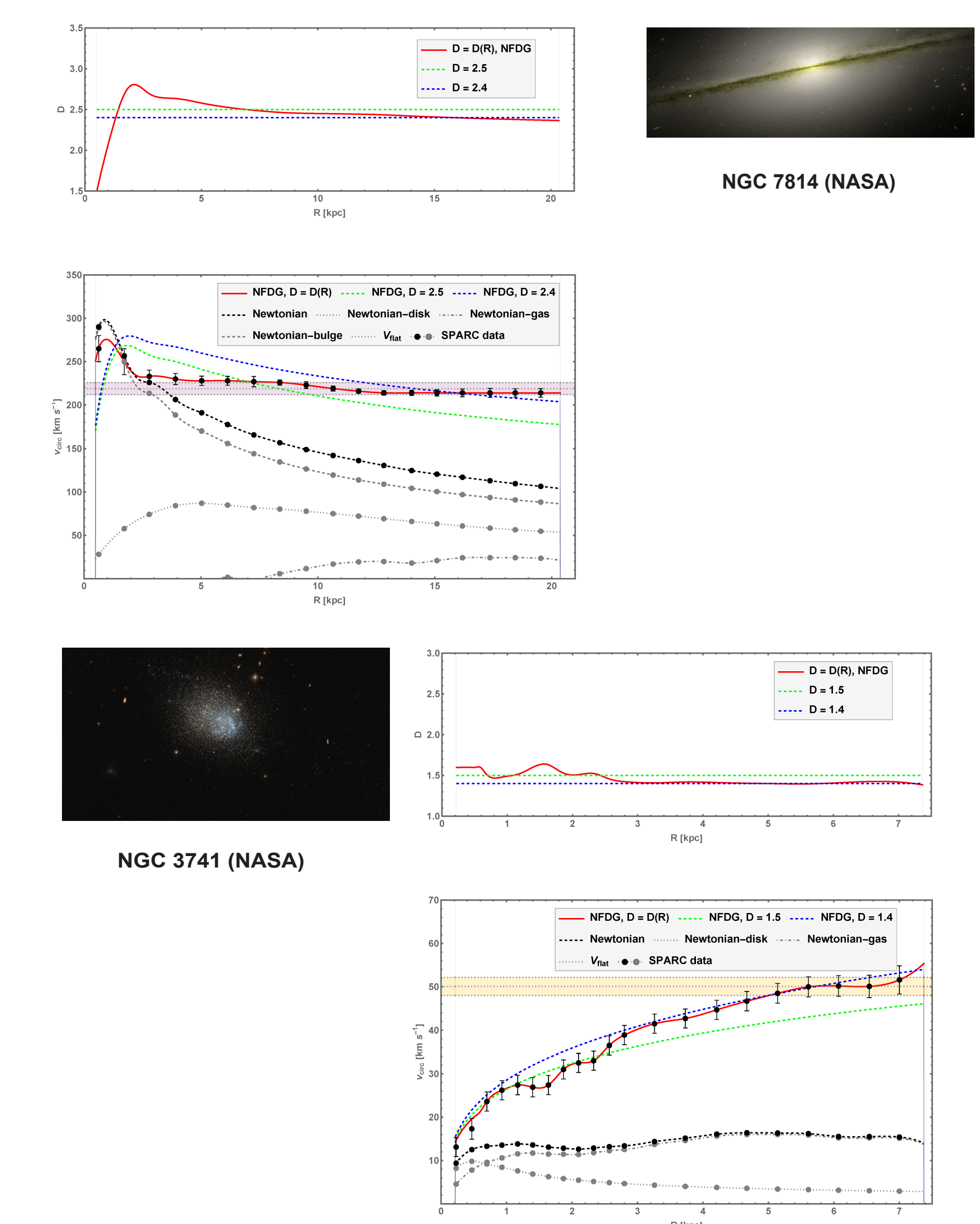
Finally, the circular velocity, for stars rotating in the main galactic plane, is simply obtained from the last equation as:

$$v_{circ}(R) = \sqrt{l_0 R |\mathbf{g}_{NFDG}(R)|},$$

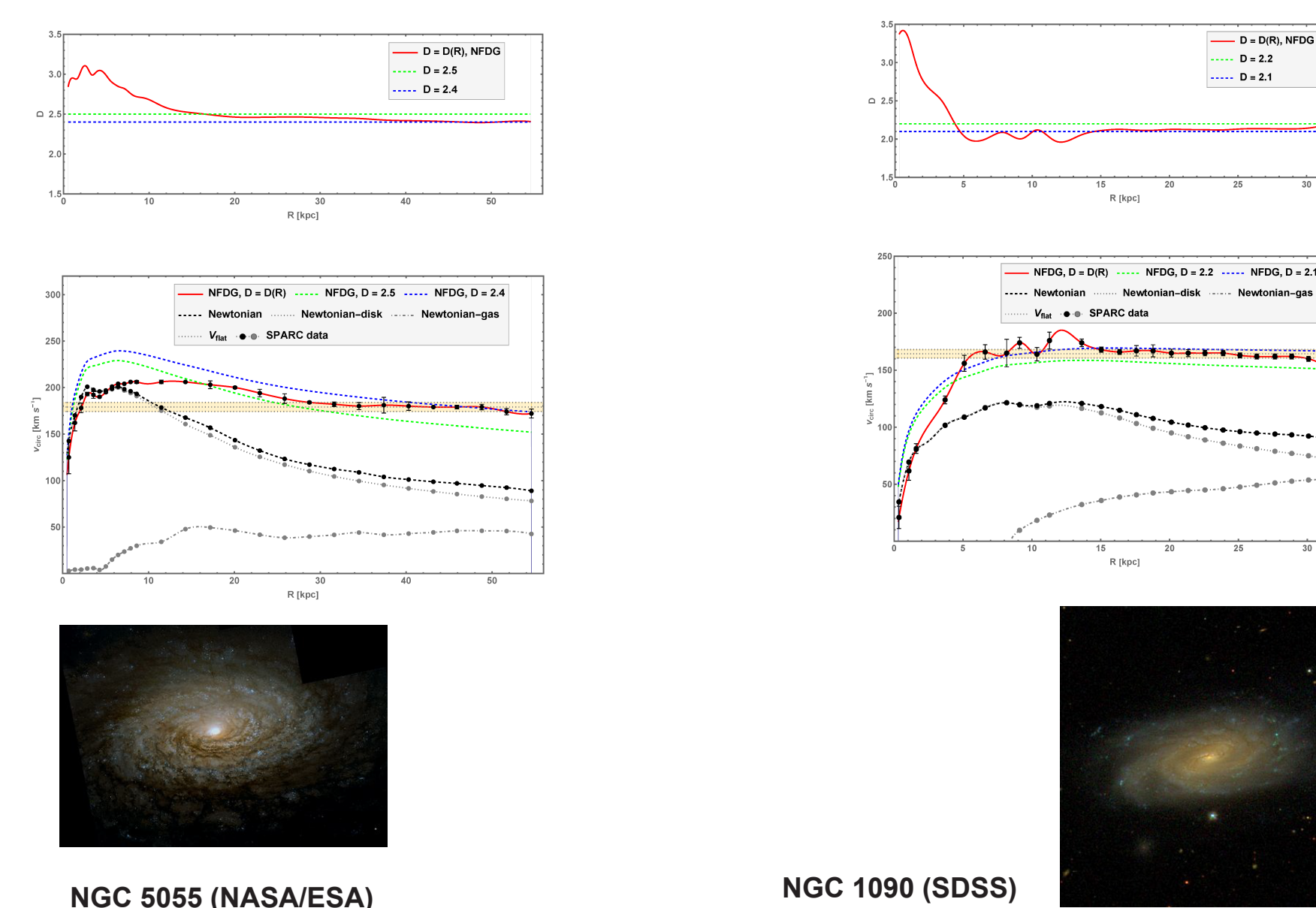
where the NFDG field is computed by using the variable dimension function $D = D(R)$ mentioned above.

NGC 7814 & NGC 3741

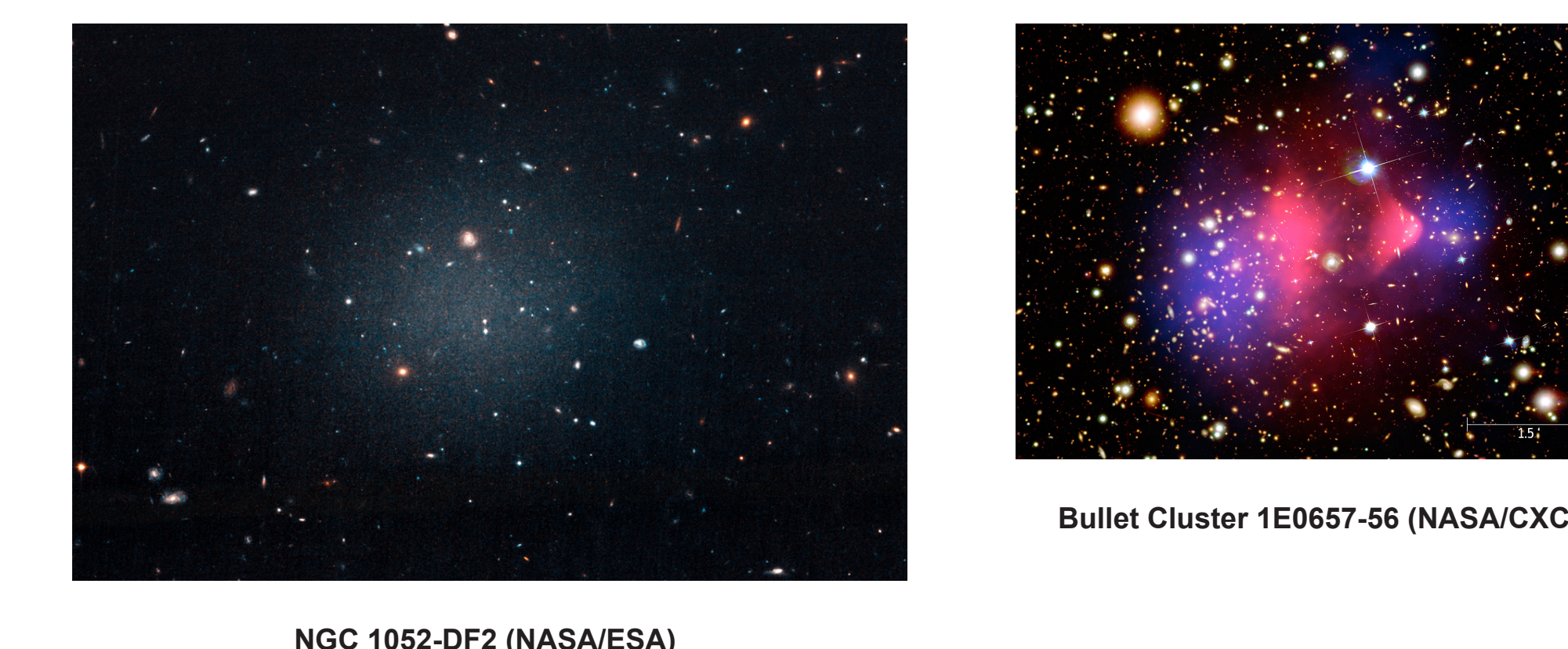
NFDG analysis of bulge-dominated spiral galaxy NGC 7814 and gas-dominated NGC 3741. Top panels show the variable dimension $D(R)$ for each case, while the bottom panels show the related NFDG velocity curves.



NGC 5033 and NGC 5055 are considered by MOND to be "golden galaxies" for the EFE to be detected, since they are located in very dense environments. On the contrary, NGC 6674 and NGC 1090 are used by MOND as perfect examples of "control galaxies" for null detection of the EFE. Our NFDG model does not expect any EFE, and no particular differences were observed, with our NFDG analysis, between these galaxies.



In the case of NGC 1052-DF2, we used arguments based on the NFDG extension of the virial theorem applied to the velocity dispersion of globular clusters. Discrepancies between observed and predicted velocity dispersions can be attributed to an overall fractal dimension $D \approx 2.9$, and not to the presence of dark matter. We also considered the case of the Bullet Cluster merger (1E0657-56), which is usually assumed to be one of the strongest proofs of DM existence. A simplified NFDG model of this collision shows that velocity discrepancies can be once again explained by a fractional dimension of the system $D \approx 2.4 - 2.5$, without using any DM.



Conclusions

In this work, we extended Newtonian gravity to spaces with non-integer dimension D . The main goal was to show that the NFDG model can be used to describe galaxy rotation curves without any dark matter components, assuming that each galaxy can be described by a characteristic variable dimension function $D(R)$.

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